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1 Introduction

Included in the collection of Pedro Nunes' works published in Basel in 1566 under the title Petri Nonii Salaciensis Opera, was the text "In Problema mechanicum Aristotelis de motu nauigii ex remis Annotatio una" (A note on the Aristotelian mechanical problem on the motion of a boat propelled by oars). This short text was never inspected by historians interested in the scientific work of Nunes. Such neglect is to be regretted because, although this brief work is, from a technical point of view, much less sophisticated than the majority of the works of the Portuguese mathematician, it is nevertheless an interesting piece of sixteenth century mathematical research with some conceptual novelties. Furthermore, since it addresses questions of a mechanical —

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1 This work is an abridged and slightly updated version of a book I recently published, in Portuguese: Henrique Leitão, O Comentário de Pedro Nunes à Navegação a Remos (Lisboa: Edições Culturais da Marinha, 2002). I want to thank Richard Barker and John Coates for helpful comments and materials sent to me on the more technical aspects of rowing. Thanks are also due to Owen Brisson for his help in improving my English. Any errors that remain are my sole responsibility, of course.

2 Petri Nonii Salaciensis Opera (Basileae: Ex Officina Henricpetrina, 1566), pp. 189-196. The text was afterwards reedited in the Petri Nonii Salaciensis de arte atque ratione navigandi libri duo (Comimbricae: In aedibus Antonij à Marijjs, 1573), pp. 121-126, and also in the Petri Nonii Salaciensis Opera (Basileae: Per Sebastianum Henricpetrum, 1592), published after Nunes' death. For bibliographical details of all these editions see Pedro Nunes, 1502-1578. Novas terras, novos mares e o que mais he: novo ceo e novas estrelas, catalogue and notes by Henrique de Sousa Leitão and Lígia de Azevedo Martins (Lisboa: Biblioteca Nacional, 2002).
2 The aristotelian Mechanical Problems

The aristotelian Mechanica — Aristotelis mechanica, also referred to as mechanica problemata, mechanicae questiones, etc. — is a work traditionally attributed to ARISTOTLE or to a member of the aristotelian school. In this text various mechanical problems are studied: the physics of levers, balances, wedges, moving objects, etc., plus some other diverse questions of a mechanical nature. The text opens with an introduction and proceeds with a discussion of thirty-five questions to each of which is provided a discussion and, as far as possible, an answer. Among these there are two questions related to the motion of boats and the action of oars. These were the two questions that served as the starting point for the analysis of Pedro NUNES in his "In Problema mechanice Aristotelis de motu nauigij ex remis Annotatio una".

In the Mechanica, the treatment given to the different questions follows a common pattern. In each question the physical properties are reduced to the properties of the lever and these are described in terms of properties of the circle. As the author of the Mechanica explains: "the original cause of all such phenomena is the circle; and this is natural, for it is in no way strange that something remarkable should follow from something more remarkable, and the most remarkable fact is the combination of opposites with each other. The circle is made up of such opposites, for to begin with it is composed both of the moving and of the stationary, which are by nature opposites to each other". The peculiarity of the circle that renders it, among all the geometrical figures, the first of all marvels, is that "it moves simultaneously in opposite directions; for it moves simultaneously forwards and backwards, and the radius that describes it behaves in the same way". The author of the Mechanica concludes: "Therefore, as has been said before, there is nothing strange in the circle being the first of all marvels. The facts about the balance depend upon the circle, and those about the lever upon the balance, while nearly all the other problems of mechanical movement can depend upon the lever."  

In the Aristotelian text the approach is dynamical, that is, the different questions are treated as problems of motion and are studied using the properties of the circular motion. This important characteristic of Aristotelian mechanics inaugurated the so-called Aristotelian tradition that differs from other mechanical traditions such as, for example, the Archimedean tradition, which analyses mechanical problems as situations of statical equilibrium. In the Aristotelian Mechanica there is not a precise distinction between the theory of equilibrium and the theory of

\[\text{made by Diego Hurtado de Mendoza in 1545 and published in the 19th century by R. Fouchér-Delbosc: Diego Hurtado de Mendoza, "Mechanica de Aristotiles [1545]", Revue Hispanique, 5 (1898) 365-405.}\]

\[\text{\textsuperscript{5}ARISTOTLE, "Mechanical problems", HETT trans. p. 333.}\]

\[\text{\textsuperscript{6}ARISTOTLE, "Mechanical problems", HETT trans. p. 335.}\]
motion; the two situations are not treated separately. It is implicit that when there is no motion, there is equilibrium. Although the much more rigorous mathematical works of ARCHIMEDES would turn out to be the most influential in the establishment of modern mechanics, it has been suggested that it was precisely the exploratory and tentative nature of the Aristotelian approach the source of its interest and fecundity. The Aristotelian Mechanica is probably the first text of Statics and it was one of the most influential texts in the development of this discipline. Besides the direct transmission of the text itself, its influence can be detected indirectly in many other works; it thus contributed decisively to the establishment of mechanics as an autonomous and rigorous scientific discipline.

Historians have differed in the assessment of the real impact of this text during Antiquity and the Middle Ages. This discussion, however, is not relevant for our present purposes since what is beyond doubt is that its major impact was felt in the sixteenth century when the text entered the world of Renaissance Europe. With the Italian Renaissance and its surprising overlap of interests between humanists and mathematicians, the Mechanica underwent an extraordinary dissemination. In many of the great renaissance libraries in Italy there were manuscript copies and already in the fifteenth century Johannes REGIOMONTANUS (1436-1476) had the intention of translating into Latin and publishing the Greek manuscript he had obtained most probably from the library of cardinal BESSARION. As is natural, with the advent of printing the text had a much wider dissemination and finally entered the learned circles of Europe.

The Mechanica was first printed in Greek in the famous editio princeps of the Aristotelian corpus compiled by BESSARION and published in the house of Aldus MANUTIUS. The text became, according to an historian, "perhaps the most influential work on mechanics in the sixteenth century." The first Latin translation was made by the Italian humanist Victor FAUSTO, published in Paris in 1517. A second translation was made by the famous editor of Aristotelian texts Niccolò LEONICO TOMEO, who published it for the first time in 1525, adding some commentaries. The several editions of LEONICO TOMEO's translation were the best-known editions of the Mechanica in the sixteenth century. Some years later the famous humanist Alessandro PICCOLOMINI popularized the text in a paraphrase, published in Rome in 1547 and afterwards in Venice in 1565.

The well known Spanish humanist and bibliophile Diego HURTADO DE MENDONZA, ambassador of CHARLES V in Venice and Trento during the years of 1539 to 1546, had an interesting relation with the text. It was he who urged PICCOLOMINI to write his paraphrase and MENDONZA himself translated the text from Greek to Spanish, a translation, however, that remained in manuscript. Several other authors included discussions of the Mechanica in their works. One such case was Girolamo CARDANO, who, in his Opus novum de proportionibus (1570), specifically addressed, among others, the questions pertaining to oars.

Following the availability of Latin printed versions the Mechanica was studied in a much wider context. In the second half of the sixteenth century...
century the *Mechanica* became part of university curricula in some universities, especially in Italy. In Padua the text was studied in the courses taught by Pietro Catena around 1570, by Giuseppe Moletti in 1581, and by Galileo in 1598. In Paris, it was discussed in the classes of Petrus Ramus around 1565. As we will see later, the comparison of these dates with the ones relative to the teaching of the *Mechanica* in the classes of Pedro Nunes — in the early thirties — will show that Nunes should be considered as one of the pioneers in the introduction of the *Mechanica* in sixteenth century Europe.

The facts described in the preceding paragraphs deserve some attention for they point to a pattern of dissemination that needs to be kept in mind when studying the contributions of Pedro Nunes. The first appearance of the *Mechanica* in Renaissance Europe was intimately associated with the activities of men who were above all humanists and *literati*. Their interest in the text was most of all related to the effort of recovering the texts of antiquity as far as possible in their pristine form. Whereas some of these men showed interest in technological and scientific questions none can be properly described as a competent scientist or mathematician. This can be appreciated by a brief inspection of the biographies of those responsible for the first Latin editions of the *Mechanica*, that is, Fausto, Leonico Tomeo and Piccolomini.

Victor, or Vittore, Fausto (1480-1551?) was a Venetian humanist with a strong interest in mechanical issues. He was a well known person in his time to the point of being mentioned in Ariosto’s *Orlando Furioso*. After some voyages in Europe, where he learned shipbuilding techniques, in 1526 he designed a quinquereme for the Venetian government. Cardinal Pietro Bembo was a great admirer of this work. The interest of Fausto in the *Mechanica* is understandable, but his edition is simply a translation into Latin without commentaries. Niccolò Leonico Tomeo (1456-1531) was a professor of philosophy at Padua University between 1497 and around 1509. He was also a teacher of Greek at Padua in the years 1504 to 1506. Although his work displays an evident interest in the study of the physical world, he was mostly noted as an expert in Greek. Besides the *Mechanica*, he also wrote widely known commentaries to several of the aristotelian *libri naturales*. In his edition of the *Mechanica*, Leonico Tomeo added some commentaries of reduced scientific value. Alessandro Piccolomini (1508-1579) was a teacher of moral philosophy in Padua and Siena. Giuseppe Moletti describes him, probably with some exaggeration, as “very well learned in peripatetic philosophy and mathematics”. The commentary of Piccolomini on the *Mechanica* was published in Rome in 1547 and afterwards in Venice in 1565. It was translated to Italian by Vannoccio Biringuccio in 1582. Piccolomini’s text — a paraphrase of the original — was widely known, but is not very innovative. Piccolomini classifies mechanics as a contemplative and mathematical discipline, that is, non-practical; similar thus to disciplines such as optics or astronomy.

Thus, until mid sixteenth century the *Mechanica* attracted especially the attention of humanists and *literati* and the recovery of the text was essentially a linguistic and philological problem. There are indications that some men of high scientific competence were interested in the text — such as Regiomontanus in the 15th century, or Tartaglia, in his *Quesiti*, of 1546 — but in general the text was part of the world of mathematics and philosophy. The main influence on the *Mechanica* was certainly that of the *Elements* of Euclide, and the law of gravity, not yet discovered by Newton, was a matter of controversy and discussion.

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13 On the teachers of mathematics at Padua, see Paul Lawrence Rose, “Professors of mathematics at Padua University 1521-1588”, *Physis*, 17 (1975) 300-304; Antonio Favaro, “I lettori di matematica nella Università di Padova”, *Memorie e Documenti per la Storia dell’Università di Padova*, 1 (1922) 3-70.

14 Victor Fausto is mentioned by Ariosto in *Orlando Furioso*, canto 46, stanza 1-7.


humanists. Fausto, Toméo and Piccolomini were interested in the Mechanica but they did not bring any innovation from the mathematical or scientific point of view. This pattern would change significantly only in the second half of the sixteenth century and in the seventeenth century, when the scientific content of the Mechanica became the focus of attraction to the text. It was only then that the men using the text used it as a scientific text which was crucial to understand, to correct, to amplify, and to insert in the tradition of mechanical studies.

It is beyond the purposes of this paper to describe in detail the results of the work of men such as Niccolò Tartaglia (1500-1557), Pietro Catena (1548-1576) or Giuseppe Moletti (1531-1588), exploring many suggestions and concepts present in the Mechanica. Suffice it to say that these works laid the foundations of mechanics as we know it today\footnote{A general study with a large selection of sixteenth century texts on mechanics is Stillman Drake and I. E. Drabkin (Eds.), Mechanics in sixteenth-century Italy. Selections from Tartaglia, Benedetti, Guido Ubald, and Galileo (Madison: The University of Wisconsin Press, 1969). More specifically on Tartaglia, Catena and Moletti, see the following: Arnaldo Masotti, "Tartaglia, Niccolò", Dictionary of Scientific Biography, (New York: Scribner, 1970-1980), Vol. 9, pp. 258-262; Giulio Cesare Giacobbe, "La riflessione metamecànica di Pietro Catena", Physia, 15 (1973) 178-196; W. R. Laird, The Unfinished Mechanics of Giuseppe Moletti, op. cit.}

The Italian mathematician Francesco Maurolico (1494-1575) was the first author to try to harmonize the Archimedean tradition with the Aristotelian tradition of mechanics\footnote{On Maurolico see Arnaldo Masotti, "Maurolico, Francesco", Dictionary of Scientific Biography, (New York: Scribner, 1970-1980), Vol. 9, pp. 190-194, and also Marshall Clagett, "The works of Francesco Maurolico", Physia, 16 (1974) 149-198; Marshall Clagett, Archimedes in the Middle Ages, Vol. 3, Part 3 (Philadelphia: American Philosophical Society, 1978), pp. 749-770.}. This very important effort would be completed by Guidobaldo Del Monte (1545-1607) in his celebrated Liber mechanicorum (1577), perhaps the most influential work on mechanics in its time\footnote{On Guidobaldo see Paul Lawrence Rose, "Monte, Guidobaldo, Marchese del", Dictionary of Scientific Biography, (New York: Scribner, 1970-1980), Vol. 9, pp. 487-489; Domenico Bertoloni Meli, "Guidobaldo dal Monte and the archimedean revival", Nuncius, 7.1 (1992) 3-34.}. In a sense the Liber mechanicorum is the first hint of the disappearance of the Mechanica because the treatment put forward by Del Monte is both so rigorous and so innovative that it would establish the canonical approach to mechanics for the following generations. Nevertheless, interest in the Mechanica did not cease with the sixteenth century and some editions and commentaries of great scientific value were published after that. In 1599 the Aristotelis Mechanica, Graeco emendata, Latina facta et commentaria illustrata was published by Henri de Monantheuil; in 1615, Giuseppe Biancani carefully analysed some mechanical questions in his influential Aristotelis loca mathematica; in 1621, Bernardino Baldi published the In Mechanica Aristotelis Problematum Exercitationes that follows the original text without innovation. The last important commentary to the Mechanica was the In Aristotelis mechanics commentarii (…), by Giovanni de Guevara, published in 1629.

All this effort around the mechanical questions culminated, as is well known, in the works of Galileo. The Mechanica had in Galileo the last, but possibly the most important, of its students and the relevance of the Aristotelian text in the thought of Galileo is today well known\footnote{“Galilée allait s’inspirer, pour son project intellectuel, d’une idée qu’il disait avoir trouvée dans les Mécaniques : la compensation de la force par la vitesse. Sous le nom de momento cet invariant est l’outil essentiel qui lui servirait à unifier, dans le cadre d’une science des machines élargie, les effets mécaniques, le mouvement des fluides, la percussion, la chute des corps et la résistance des matériaux”, François de Gandt, “Les Mécaniques attribuées à Aristote et le renouveau de la science des machines au XVIIe siècle”, Les Études Philosopohiques, 3 (1986), p. 392. See also Stillman Drake, "Galileo Gleanings V: The earliest version of Galileo's mechanics", Osiris, 13 (1958) 269-290; Galileo, On Motion and On Mechanics. Translated and edited by I. E. Drabkin and Stillman Drake (Madison: The University of Wisconsin Press, 1960). Galileo addressed specifically the motion of oars in a letter to Giacomo Contarini (22 March 1593), in: Le Opere di Galileo Galilei, Antonio Favaro (ed.), (Firenze: Barbera, 1968) Vol. X, n. 48, p. 57. See also the interesting work by Jürgen Renn and Matteo Valleriani "Galileo and the Challenge of the Arsenal", Preprint MPI (Berlin, 2001).}. The Mechanica is explicitly mentioned in the Discorsi, and its influence can also be seen in the correspondence of the great Italian scientist.

With Galileo's researches on mechanics the Aristotelian Mechanica was rapidly forgotten. After its introduction in Renaissance Europe at the end of the fifteenth century the text had a brief life. A brief life it
may have been, but certainly a very influential one and it is impossible to write a history of modern mechanics without reference to this work.

3 Pedro Nunes and the mechanical questions

Pedro Nunes’ “In probem a mechanicum Aristotelis de motu nauigij ex remis annotatio una”, was published in the 1566 Basel edition of his Opera. It was included in the De arte atque ratione navigandi, of 1573, and it was published again in the 1592 edition of the Opera, again in Basel. Apart from minor typographical variations and slightly different rend ofing the figures, the text of these editions is identical. [The Appendix presents a complete transcription of Nunes’ text].

Although the study of the motion of a boat propelled with oars was published only in 1566, Nunes’ interest for the Mechanica dates from much earlier. In fact, from a surprisingly early date. The dedicatory letter addressed to King João III that opens Nunes’ De crepusculis (1542) provides important elements to establish the chronology of Nunes’ interest in these questions. In this letter, when referring to the classes that King João III had ordered him to teach to D. Henrique, one of the King’s brothers, Nunes states:

Ten years ago you, O most liberal king, ordered me to teach him the mathematical sciences. Diligently and in short time he learned the Elements of Arithmetic and Geometry of Euclid, the Treatise on the Sphere, the Theories on the Planets, part of the Great Composition [Almagest] of Ptolemy, the Mechanica by Aristotle, all the Cosmography and the practice of some ancient instruments and others that I had invented for the art of navigation.21

Since this letter is dated 17 October 1541, “ten years ago” refers to

21 This quotation can be read in Portuguese in the national edition of Pedro Nunes works, Pedro Nunes. Obras, (Lisbon: Academia das Ciências de Lisboa, 1943) Vol. II, p. 149. [Henceforth this edition will be identified as Pedro Nunes. Obras.] The original Latin text is: “Eum tu rex humanissime decem abhinc annis, mathematicis scientiis instituendum a me curasti. Didicit ille diligentissime breuique tempore, Arithmetica et geometrica Euclidis elementa, Sphaerae tractatum, Theoricas plan-

events that took place around 1531. That is, the Aristotelian Mechanica was part of the subjects taught by Nunes since the early thirties. Moreover, we know that he continued to address these matters because the text of 1566 makes explicit reference to “my disciples”, that is, the students at the University of Coimbra, where he taught mathematics from 1544 to 1562.

The fact that Nunes was already teaching the Mechanica around 1531, when compared with the chronology of the dissemination of this work leads inevitably to the conclusion that he was one of the very first men to include these matters in his classes (much earlier than Ramus in 1565 or Catena in 1570) and that he was perhaps the first fully competent mathematician to have been interested by the text at a period when it was mostly under the attention of humanists.22

Besides the brief mention of the Mechanica in the dedicatory letter of 1541 in De crepusculis and the “In probem a mechanicum Aristotelis de motu nauigij ex remis annotatio una”, published in 1566, Pedro Nunes makes further mention of the Mechanica in one other of his works. In the Libro de Álgebra (1567) he includes some passages that denote the influence of the Aristotelian text and, more specifically, in the first chapter he writes:

O quan bueno fuera, si los Autores que escriuyeron en las sciencias Matematicas, nos dexaran escriptos los sus inventos por la misma via, y con los mismos discursos que hizieron, hasta que pararan en ellos. I no como Aristoteles dize en la mecanica de los artifices, que nos muestran dela machina que tienen hecha la de fuera, y esconden el articifio, por parescer admirables.23

The passage refers to a part of the Mechanica and its tone reveals
that Nunes presumed his audience to be fully acquainted with the Aristotelian text. In this same Libro de Algebra there is one other passage worth quoting where the Mechanica is mentioned:

Y no es de creer Hieronymo Cardano, el cual dize en libro de las subtilizas que Iordano no demonstró lo que del auemos allegado, antes lo que mismo Cardano demuestra, de Iordano lo harto, y lo que dize declarando a Aristoteles en la mechanica, esto es de su cabeza, y es falso, vease la traducion de Victor Fausto, porque el otro interprete no sabe lo que dize.

The “libro de las subtilizas” of Cardano is obviously De Fabulis libri XXI and this passage serves also to point out that Nunes was familiar with the works of Jordanus.

What were the sources for Nunes’s commentary on the Aristotelian Mechanica? This question can be addressed in the larger context of understanding what was Nunes’ acquaintance with the different traditions and texts of mechanics, and in the more narrow scope of his immediate sources. Concerning Nunes’ interest for mechanical questions a brief summary can be provided as follows. It is known that around

principal, que es el tratado de las proporciones, y primeramente de la dificion de Propolucion”, Pedro Nunes. Obras, VI, p. 81. This excerpt was later reproduced in Latin by John Wallis who explicitly mentioned Nunes as its author: “O quam bene foret, si qui en scientia Mathematicis scripsisset authores, scripta reliquisse inuenta sua eadem modo, et per eodem discurus, quibus ipsi in ea primum inciderunt; et non ut in Mechanica loquar Aristoteles, de Artificibus, qui nobis fars ostendent suas quas fecerint Machines, sed artificium abseundunt, ut magis appareat admirabile.” John Wallis, De Algebra Tractatus Historicus et Praticus, in Operum Mathematicorum, volume II (Oxonii, 1693).

Nunes was surely using Fausto’s translation where this passage reads: “Animadvertentes igitur hanc quae circulo est naturam opifices instrumenta machinae, qua sind component atque afformant supressis principiis ita ut in machina tandem constructa id dumtaxat quem mirum est conspic posset causa vero delitescat et hoc velut praestigio ipse quoque miraculo sit”. Victor Fausto, Aristotelis mechanica (Paris, 1817), sig. A v. The modern English edition of the Mechanica is: “So making use of this property inherent in the circle, craftsmen make an instrument concealing the original circle, so that the marvel of the machine is alone apparent, while its cause is invisible”. Aristotle, “Mechanical problems”, Hett., trans., p. 337.

Pedro Nunes. Obras, VI, p. 85.

Pedro Nunes and the Aristotelian Mechanical Problems

1541 Nunes made a translation of Vitruvius’ De architecture; also, it can be ascertained that he was familiar with the works of Jordanus of Nemore, with Cardano’s De subtilitate (1551) and with Giorgio Valla’s De expetendis et fugiendis rebus (1501). Furthermore, he had carefully studied Archimedes’ works and references to them can be found in several of his books. In short, Nunes was aware of the different mechanical traditions and authors available in the sixteenth century. Concerning the more specialised question of the sources for his knowledge of the Mechanica, both in his annotation of the question of ours [see Appendix] and in the Libro de Algebra [see quotation above] he explicitly refers to the edition of Victor Fausto (Paris, 1517). In view of his command of the scientific literature of his time, and the fact that he was already using the Mechanica around 1531 it is reasonable to assume that he was also familiar with the 1525 Venetian edition of Niccolò Leonico Tomeo, but he makes no explicit mention of it.

4 Pedro Nunes’ commentary

Pedro Nunes’ commentary “In problema mechanicum Aristotelis de motu nauigij ex remis” is an example of some of the most typical characteristics of his intellectual personality. As often was the case, the initial motivation for his study seems to have been connected with his obligations, as Royal Cosmographer, tutor of princes, or teacher of mathematics at the university. Many of the problems he treated had their origin in the context of applied science and practical activities, which Nunes’ dutifully mentions. However, once the origin is specified, his treatment is theoretical, rigorous and, to a large extent, useless in the original context where the questions were formulated. In the case of the “In problema mechanicum Aristotelis de motu nauigij ex remis”, Nunes explains that his reflections originated in discussions with his students and that he includes his text just after his nautical treatises due to the similarity of subjects. This, however, somewhat eludes the point that the text is a strictly theoretical exercise with hardly any foreseeable ap-
plication in everyday life at sea. Its main purpose seems to have been to clarify the complex question of the mechanics of rowing and to criticize the arguments of Aristotlē. The text is a theoretical exercise in Euclidean geometry with some important insights into the physics of oar motion. Nunes ignores all technological or practical problems associated with rowing and abstracts the whole problem to one of properties of plane figures, straight lines, angles, etc. He also ignores those aspects that in present-day language one would call dynamical (i.e. those that consider forces and the cause of motion), to concentrate solely on the kinematical description.

In the following I will briefly analyse the text of Pedro Nunes. The complete transcription of the original Latin is in the Appendix. I take some liberties of language and omit the explanation of the propositions of Euclid's Elements since these have to do with the simplest properties of triangles. The diagrams were redrawn to enhance comprehension.

Rowing and the motion of oars is not a trivial physical problem because it has to do with the motion of an element (the oar) that is fixed to a reference frame (the boat) that it is also moving. Thus, although the oar is simply acting as a lever, some care must be taken because this lever — that is producing the motion — is itself in motion. A further slight complication arises because although the boat moves in a straight line, the motive force, i.e. the hands of the rower, acts along a circular path. Care must be exercised when classifying what type of lever the oar is. The analysis of rowing in the Aristotelian text suffers from several problems due to an incorrect consideration of these subtleties. For example, there is some ambiguity in the type of lever to consider. The author of the Mechanica starts by saying that “the thole-pin is the fulcrum (for it is fixed), and the sea is the weight, which the oar presses; the sailor is the force which moves the oar” — that is, he classifies the oar as lever of the first class ("vектe primi generis"). Thus, to the author of the Mechanica, in each stroke, the end of the oar inside the boat (in the rower's hands) and the oar blade in the sea move in opposite directions, both of them turning around the thole-pin. However, afterwards, faced with the problems of such a judgement, he will not be consistent with such description. Furthermore, the author of the Mechanica states that the boat moves forward a distance always greater than the distance the oar blade traverses in the opposite direction. A further difficulty with the Aristotelian analysis of the oar motion has to do with its qualitative tone, with only a very sketchy and imprecise mathematical treatment. As we will see below, both from the conceptual point of view and from that of the mathematical treatment used, Nunes' commentary is a major step forward in the correct description of rowing.

Pedro Nunes starts his analysis of the boat motion saying that he wishes to clarify the "obscure" arguments of Aristotlē. Nunes' starting point is the figure and the nomenclature used by Victor Fausto (Fig. 1) 28. In this figure, the oar is represented by the line segment ab, with a the oar handle and b the oar blade. It is assumed in this figure and in all the following demonstrations that the thole-pin c, is at the middle of the oar, that is, at the mid point of the line segment ab. Nunes starts to show that if the oar — initially along ab — will have position de after the stroke, than the thole-pin, and also the boat, where the tholepin is fixed, will not move. The demonstration is simple. It is therefore proved that if after the stroke the distance traversed by the oar handle and the oar blade are identical the thole pin, and hence boat, will not move.

28This figure is also presented in Leonico Tomeo's Conversio mechanicarum Quaestitionum Aristotelis cum figures (Paris, 1530), p. 33, and in Piccolomini's paraphrase, Alexandre Piccolomini in mechanicas questiones Aristotelis, paraphrasis paulo quandem plenior (Rome, 1547), fol. xxvii, but Pedro Nunes never quotes these texts and I presume he has not used them.
In the aristotelian text, to which FAUSTO had added figures, it had already been shown that in each stroke the distance traversed by the oar handle is greater than the distance traversed by the oar blade. NUNES repeats that demonstration which again offers no difficulties and it is based in the most simple properties of triangles. Fig. 1 is used again. It is assumed that, after the stroke, the oar is not in position \( de \), but in position \( dz \), which intersects \( ab \) in \( t \). Since angle \( cad \) is equal to angle \( cbe \) (internal angles), and angle \( atd \) is identical to angle \( btz \) (opposite angles), then it follows that triangles \( atd \) and \( btz \) are equiangule. By proposition 4 of Book VI of Euclid’s Elements, their sides are proportional and one has in particular \( at/bt = da/bz \). Since, by construction, \( at > bt \), one concludes that \( da > bz \), that is, the oar handle is displaced a longer distance than the oar blade — precisely what ARISTOTLE had stated.

Pedro NUNES then slightly improves the analysis of Victor FAUSTO, by pointing out that Figure 1 is not correct. In fact, when the oar has position \( dz \), its extremity will not be exactly in \( z \), but slightly upwards. He demonstrates that segment \( ab \) is greater than \( dz \) and therefore, at the end of the stroke, the oar blade has to be beyond \( z \). He presents the more correct Figure 2:

In this figure \( k \) denotes the position of the oar blade at the end of the stroke. Therefore, the space traversed by the oar blade is not \( bz \), but \( bk \). NUNES shows that with this more correct depiction it is still true that the oar handle traverses a longer distance than the oar blade, that is, that one has \( ad > bk \).

Up to this point NUNES is analysing the kinematics of rowing using the same notions previously used. He will now introduce important conceptual precisions which denote a careful reflection upon the question of rowing. He starts by noting that the movement of the oar handle is in reality composed of two motions: the motion around the thole-pin — which NUNES will henceforth call “proper motion” — and the motion which results from the fact that the boat itself is moving.

Composition of motion was a topic considered in the tradition of Aristotelian mechanics, but had not been applied to the description of rowing. Furthermore, NUNES will consider a composition of two different types of motion: a circular and a rectilinear one. In the Mechanica
an analysis of the circular motion is provided and it is asserted that "the radius describing the circle is performing two motions". In the text the analysis starts by considering a body affected by two motions that have a constant ratio — that is, as we would say in present day terminology, a body whose velocity can be decomposed into two perpendicular components that have a constant ratio. In this case, the author of the Mechanica correctly says: "Whenever a body is moved in two directions in a fixed ratio it necessarily travels in a straight line, which is the diagonal of the figure which the lines arranged in this ratio describe" 29. He then shows that "if a body travels with two movements with no fixed ratio and in no fixed time, it would be impossible for it to travel in a straight line" 30. The Aristotelian author then considers motion in a vertical circle, such as, for example, that of a body falling along a circular trajectory. It is implicit in this case that one needs to consider the vertical motion of fall and the horizontal motion that keeps the body in its circular path. He concludes: "This happens with any radius which describes a circle; it moves along a curve naturally in the direction of the tangent, but is attracted to the centre contrary to nature" 31.

The analysis of Pedro Nunes goes somewhat further since it considers two different types of motion: a circular one around the thole-pin, and a rectilinear one associated to the motion of the boat. By clarifying that in the analysis of rowing one has to deal with these two motions — i.e., in modern parlance, that the frame of reference in which the oar is moving is itself in motion — Nunes' approach is clearly an important step forward in the comprehension of the mechanical problem of rowing.

With this important notion in mind, Pedro Nunes characterizes the kinematics of our motion in a set of five propositions rigorously demonstrated.

In the first proposition he establishes that "if the rowers move the boat, then the oar handle will always traverse a longer distance than the boat". It is interesting to note that this first proposition immediately centers the discussion in a comparison of distances traversed by the motive force (acting on the oar handle) and by the boat. Indeed such is the correct way to proceed if one wishes to evaluate rowing as a simple machine. Nunes therefore abandons the previous approaches and presents a new diagram, Figure 3.

![Figure 3](image)

In this figure $ac$ is the oar position before the stroke; $a$ is the handle, $b$ the thole pin and $c$ the oar blade. After the stroke the oar position is $ef$. The thole-pin, and therefore, the boat, was displaced by the amount $bd$. The oar handle went through the path described by the curved line $ae$. The horizontal projection of $ae$ is the segment $az$, and Nunes will use this horizontal projection to estimate the distance traversed by the handle. The demonstration of the first proposition consists simply in showing that $az$ is larger than $bd$. The demonstration is very simple. Since triangles $agz$ and $bgd$ are equiangle, their sides are proportional. Thus, $ag/bg = az/bd$. Now, since $ag > bg$ it follows that $az > bd$, that is, the oar handle traverses a larger distance than the boat, as one

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wanted to demonstrate.

Figure 3, however, does not clarify the central aspect of Nunes' approach to the problem, that is, the decomposition of the oar motion into two motions. He then presents another figure (Fig. 4), which clearly separates the "proper motion" of the oar around the thole-pin from the boat motion. In Figure 4 it becomes clear that the total space traversed by the handle, ae, is the composition of two displacements: its "proper motion" around the thole-pin — corresponding to the curved line ah — and the motion of the boat. Nunes quantifies these two displacements by their horizontal projections. Thus, the total space traversed by the handle is measured by the line segment az. This segment is the sum of ai — horizontal projection of the curve ah — and the segment iz, that measures the boat displacement. Evidently, iz = bd.

The second proposition states that "if the handle in its proper motion and the boat are displaced by the same amount, it is impossible for the oar blade to move". This proposition directly refutes Aristotle's opinion that, by interpreting the oar motion as the motion of a lever of the first kind affirmed that each time the boat was moved forward the blade was moved backwards. Nunes demonstrates this proposition in two different ways. First by reductio ad absurdum, and then in a direct manner. The demonstration by reductio ad absurdum uses Figure 5.

The condition is that ak = bg. Nunes will then show that if is assumed that the oar blade describes a (non-zero) arc cd, whose horizontal projection is cz, an absurd conclusion will follow. Thus the supposition is false and the conclusion is cz = 0. Once again the demonstration is simple and proceeds according to the following steps: first it is shown that the equiangle triangles ebc and kba are identical (it must be kept in mind that it is always assumed that the thole-pin is placed at the middle of the oar). Thus ak = ec. Using the hypothesis (ak = bg) it follows ec = bg. But, by construction, ez = bg. From this follows the absurd conclusion that the part (ez) is equal to the whole (ec). Therefore, the hypothesis that cz is different from zero must be false. The oar blade must remain immobile.
The same proposition is demonstrated directly, using Figure 6, a process that NUNES thinks to be more evident. The demonstration practically reduces to the drawing of the figure since all the demonstration steps are elementary. In this figure ac represents the oar at the beginning of the stroke and et the oar at the end. By construction ge = ab (therefore the thole-pin b is transported to the straight line et, but will stay below the point g). The line segment bg measures the displacement of the boat. The condition of the proposition is that ak = bg. It should be noted, as explained by NUNES, that by the statement “the oar blade does not move” it is meant that it has no motion in the direction of the boat displacement, that is, according to Fig. 6, the oar blade does not move along the horizontal direction. In fact, with the oar stroke, the blade moves from c to t, but this displacement is perpendicular to the direction of the boat movement and does not affect the demonstration.

In the third proposition NUNES demonstrates that, if the distance traversed by the oar handle in its “proper motion” is twice the distance the boat traverses, then this path made by the boat is equal to the distance traversed backwards by the oar blade. This proposition, similarly to the previous one, explicitly contradicts the Aristotelian statement which asserted that the oar blade always traversed backwards a distance smaller than that traversed by the boat. NUNES’ demonstration uses Figure 7. The initial condition of the demonstration is that ae = 2bd. The demonstration consists in showing that with this condition, ch = bd. NUNES starts by establishing that triangles aeb and cbz are identical and thus ae = cz. Furthermore, by construction, bd and hz are identical and, by hypothesis, ae = 2bd. Therefore it immediately follows that ch = bd.

NUNES demonstrates also the reciprocal proposition, namely, that if the boat traverses a distance identical to the distance traversed backwards by the oar blade (bd = hc), then the oar handle will traverse, in its proper motion (ae) a distance double to that traversed by the boat (ae = 2bd).

Figure 6

Figure 7
The fourth proposition establishes that “if the boat moves less, but still more than half the distance moved by the oar handle, then it will move a greater distance than the one traversed backwards by the oar blade”. The demonstration uses Figure 7. The demonstration is completed with a corollary and the demonstration of the reciprocal proposition. All the constructions and demonstrations use arguments similar to the ones employed so far and require no further explanation.

In the fifth and last proposition, Pedro Nunes analyses a particular situation. He demonstrates that “if the boat is displaced more rapidly than the oar handle, the blade will move forward, never backward, and it will traverse a space identical to the excess of space traversed by the boat in relation to the space traversed by the handle”. This situation is represented in Figure 8, where the movement of the boat, $bd$, is greater than the “proper motion” of the handle, $ah$. Nunes analyses this case and concludes that such a motion cannot be due only to the oars and that other forces (wind, currents, etc.) must be at play.

Having established these five propositions, Pedro Nunes is aware of having performed a more rigorous study than the one presented in the two questions of the Aristotelian Mechanica. He then concludes his analysis with a hard critique directed against Aristotle, referring to the ambiguity of his questions and the ignorance revealed in his answers: “Ex his theorematis liquet quam incerta interroget Aristoteles, et quam inscrite respondeat”. These words reflect not only the hubris of an accomplished mathematician, sure of the strength of his analysis. They are also a clear statement that, in mathematics, nothing, not even the respect that is due to the immortal name of previous thinkers, is more important than a precise and correct demonstration.

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To end our analysis of Nunes’ text some brief words about the dissemination of his work are necessary.

One can start with a speculation related to the possible influence of Nunes’ interest in the Mechanica on his correspondents, in particular John Dee. We have no definite information. However, from the known fact that Dee had a profound admiration for the work of Nunes, and that the famous English mathematician also showed an interest in mechanical questions, one can speculate that Nunes’ text was known and appreciated by him. It should be kept in mind that mechanical questions and vitruvian themes were important scientific interests of John

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32 The important relations between Nunes and Dee would require a separate study. The amiable nature of that relation and the profound admiration that Dee nurtured for the Portuguese can be gauged by the letter that John Dee wrote to Gerard Mercator, on July 20, 1558. “And if my work cannot be finished or published while I remain alive, I have bequeathed it to that most learned and grave man who is the sole relic and ornament and prop of the mathematical arts among us, D. D. Pedro Nunes of Salacia, and not long since prayed him strenuously that, if this work of mine should be brought to him after my death, he would kindly and humbly take it under his protection and use it in every way as if it were his own: that he would deign to complete it in every way as if it were entirely his. And I do not doubt that he will himself be a party to my wish if his life and health remain unimpaired, since he loves me faithfully and it is inborn in him by nature, and reinforced by will, industry and habit, to cultivate diligently the arts most necessary to a Christian state”, John Dee on Astronomy. Propaedeumata Aphoristica (1558 and 1568), edited and translated with general notes by Wayne Shumaker, with introductory essay by J. L. Heilbron (Berkeley: University of California Press, 1978), pp. 114-115. The Latin text had previously been published by E. G. R. Taylor, Tudor Geography, 1485-1583 (London: Methuen, 1930), pp. 257-8.
It is thus possible that questions related to mechanics had been discussed between the two men.

The critique that Nunes addresses against Aristotle links him to Tartaglia and Benedetti who were also critiques of the Mechanica. The former conducted his studies without the influence of Nunes, but one may admit that the latter knew the commentary of Nunes since he was aware of other works of the Portuguese.

Leaving aside these somewhat speculative pursuits, it is possible to identify some authors who have studied and reproduced Nunes text. In the final year of the sixteenth century Henri de Monantheuil published in Paris the learned Aristotelis Mechanica, Graeca emendata, Latina facta et commentariis illustrata ab Henrico Monanthelio, (Paris, 1599) 34. Monantheuil reproduces almost integrity the text of Nunes from pages 82 to 90. He analyses the demonstrations of the Portuguese, completing them in some passages and reproduces the figures adding one more for ease of comprehension. The observations of Monantheuil are worth a more detailed study which, however, I will not do here. It suffices to point that he clearly perceived the importance of Nunes’ approach for he attributes to the Portuguese mathematician the statement that Aristotle’s problems in his analysis stem from the inability to perceive the different motions at stake, namely, the proper motion of the oar and the oar motion in consequence of the boat’s movement 35.

Perhaps of even more significance than the work of Monantheuil is the fact that the Jesuit Giuseppe Biancani turned to Nunes’ text while studying the questions related to oar movement in his influential Aristotelis loca mathematica (1615) 36. The mechanical questions are not the only occasion for Biancani to refer to Nunes’ works and one finds for example some references also to Nunes’ De crepusculis 37. Biancani’s attention to the works of Nunes is one more confirmation of the important influence the Portuguese mathematician had in the circles of Jesuit mathematicians. As the historian Ugo Baldini recently pointed out, “Tra i matematici del medio Cinquecento Pedro Nuñez fu uno dei più influenti sulla scuola di Clavio” 38. In the Aristotelis loca mathematica the mechanical questions are inspected from page 145. In the quaestio quarta (“De Remo”) and quaestio quinta (“De Temone Nauis”), he studies the motion of oars and rudder. Biancani starts by calling attention to the fundamental error made by Aristotle — and also by Piccolomini — by describing the oar as a lever of the first class 39. He then proceeds to study the two questions, but his discourse is interrupted to give way to Nunes’ text. From page 162 to page 168 the very subtle ideas of the very sharp mathematician are reproduced in full 40.


34Rose and Drake observe: “On the whole, Monantheuil’s is the most complete and erudite of the sixteenth-century commentaries on the Mechanica, though not the most original in outlook”. Paul Lawrence Rose and Stillman Drake, “The pseudo- aristotelian Questions of Mechanics in Renaissance culture”, op. cit., p. 158.

35“Atque ex his theorematibus concludit Nonius Aristotelem confusum in quaestione hoc problematicum, cum non distinguere inter motum remi proprium, et motum a navi translatum et advenientem”, Henri Monantheuil, Aristotelis Mechanica, p. 90.

36Aristotelis Loca Mathematica. Ex universis ipsius Oepubris collecta, & explicata (…) Authore Iosepho Biancano Bononiensi & Societate Jesu, Mathematicarum in Gymnasio Parmensi Professore. Bononie, MDCXV. Apud Bartholomeum Cochim. Giuseppe Biancani (born in Bologna, 1565, admitted to the Society of Jesus in 1591) played an important role in the scientific scene in the early seventeenth century. His Sphaera Mundis seu Cosmographia demonstrativa (…), (Bononie, 1620), heralds the “official” adoption by the Jesuits of Tycho Brahe cosmological model. Besides his participation in astronomical and cosmological debates he was also one of the most noted protagonists of the debate known as the quaestio de certitudine mathematicarum.

37For example, while discussing the question “De altitudine montis Caucasi”, Biancani uses an argument that makes use of the duration of twilight and he informs (p. 99) that “ut paet ex tabula quantitatis Crepusculi, quae est apud Nonium et apud Claudio in sphaera ultimae editionis; quae quantitates reputari geometrico calculo potest, ut docet Nonius, Claudius, et Maginus (…).”


39“Et, qui superiora intellecet satis clara videtur. Illud tamen non omissum, scilicet dicendum potius Remum esse vectem secundis generis, quam primi, quod forte Arist. non animaduerit, nec Piccolomineus, nam mare est hypomocion, respectu enim nauis non movetur, sed manet, scilicet autem simul cum tota navi est pondus motuum; vere enim navis ipsa movetur, mouens est ipse remex. Reliqua in textu sunt clara”. Biancani, Aristotelis loca mathematica, p. 159.

40Qua propter operaepretium me factum existimo, si Petri Nonij acutissimi...
Curiously the admiration of Biancani for the text of Nunes would be the cause of some upsets to the Jesuit. In the evaluation made to the Aristotelis loca mathematica, the censor, Joannes Camerota, although indicating that the book could be printed, advises some changes. Among the changes mentioned it is explicitly stated that the hard critique Nunes had addressed to Aristotle, Biancani had transcribed, should be omitted.41

In 1629 Giovanni di Guevara published in Rome his In Aristotelis mechanicas commentarii (...), which includes a discussion of the questions about oar motion. Guevara cites Biancani and Baldi and, following a recommendation others had already expressed, directs the reader to the propositions “quae Petrus Nonius acutissime demonstrat in sua Annotatione” 42.

All evidence leads us to conclude that from the end of the sixteenth century to the early decades of the seventeenth century the annotation of Pedro Nunes was an important text for those interested in the mechanics of rowing. In the second half of the seventeenth century the questions

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41 Joannes Camerota advises Biancani to show more respect for other authors, “Primum in generem momentar Autor ne scriptores sive antiquos, sive modernos aut carpat, aut vituperet; praesertim si catholici sint, et alculus nominis”. He underlines that “Multa magis id obser tem in philosophis celebritibus, praesertim vero Aristotele. Quem si quando confutare cogatur, faciat id modeste; quod si potest, defendat.” Following these indications, he lists the passages where Biancani had failed to show the appropriate respect for Aristotle, explicitly mentioning Nunes’ text that he had transcribed: “[...] et num. 229 sub fine propos. 5 pag. 127: “Ex his theorematibus — inquit — liquet quam incerta interroget Aristoteles, et quam incite respondeat” etc. Quod si haec postrema verba non sunt Autoris, sed Petri Nonii, id ipsum eo loco Author [admonet] sic nimirum loqui Nonium”. Cf. Ugo Baldini, Legem Impone Subactis (Roma: Bulzoni, 1992), p. 229.

42 Giovanni di Guevara, In Aristotelis mechanicas commentarii, una cum additionibus quibusdam ad eandem materiam pertinentibus (Romae: Apud Iacobum Mascardum, 1627 [published only in 1629]), p. 112.
APPENDIX

Petri Nonii

In Problema mechanicum Aristotelis de Motu navigii ex remis Annotatio una.

Cum olim discipulis nostris mechanicas Aristotelis quaestiones interpretemur, nonnulla circa problema illud annotauimus, cur magis procedat nauigium, quam remi palmula in contrarium. Aristotelis enim ratiocinatio obscura est: quam nos tamen ut alicujus lucis haberet, ad hunc modum explicauimus; & propter materiae similitudinem hisce nostris libris de Nauigandi ratione adiunximus. Supponit autem ipse autor remi palmulam retrocedere, quoties nauigium in anteriora progreditur, locumque Scalmi super quo circulari motu remus vertitur, in medio ipsius remi positum esse, ut scilicet tantum distet a manubrio, quantum a palmula. Duæ itaque lineæ ponuntur æquales ab & de, quæ quidem in c, puncto medio se inuicem secunt, & connectantur da & be: remus autem in initio unius remigationis positionem habeat rectam lineam ab, sitque a manubrium, b palmula, c vero scalum. Cum igitur a, remi caput in fine ipsius remigationis eo translatum fuerit ubi d, non erit b ubi e. Si enim ibi fuerit remus igitur positionem habebit rectam lineam de & quoniam contrapositi anguli qui ad c æquales sunt, & duo
Nam quoniam duo latera bd & dk, trianguli bdk, duobus lateribus bd & de, trianguli bed aequalia sunt, sed minor est angulus bdk angulo bdc: minor igitur erit basis bk base be, per vigesimam quartam primi, quod demonstrandum erat.

 прястра quod Aristoteles ratiocinando sumit, tantum spatium configere nauigium, quantum remi manubrium, ambiguum est. Nam remi manubrium duabus fertur motionibus: una propria circularique super Scalmo: altera vero, qua una fertur cum ipso nauigio. Spatium igitur quod omnino decursum est a remi manubrio, eo quod a nauigio confectum est, maius erit. At si paria spatia decursa esse intelligat a remi manubrio motu proprio, & a nauigio, neque hoc difficultate caret. Nam nauigium interdum maius spatium percurrret, interdum minus, iuxta remigium vires, & prout mari remi palmula immersa fuerit: remi vero manubrium tametsi ab exiguis viribus moeauetur: haud minorem tamen ambitum describere, quam si a multo maiore virtute moeuret. Quapropter ut huiusmodi Aristotelis sententiam examinaremus, Theoremata quae sequuntur, demonstrauius.
Propositio prima.

Si Remiges nauigium mouere possunt, maius semper spatium remi manubrium percurrit, quam nauigium.

Sit enim remus ac, manubrium a, scalmus b, qui propter nauigii motum spatium percurrat a b in d, in quo loco ipse remus ac, situm rectitudinis habeat e f. Spatium itaque quod a conficit, curva linea sit ac, cui recta linea respondeat az, in rectam ef perpendicularis.

Nauigium vero idem spatium conficit, quod scalmus b: aicigitur ipsum az, rectam lineam recta bd maiorem esse. Secet enim recta ac, rectam ef in g: æquangula sunt igitur bina triangula agz & bgd: quapropter sicut ag ad bg, sic az ad bd, per quartam sexti libri Euclidis: maior est autem ag ipsa bg: & maior igitur erit az, quam bd, & proinde maius spatium remi manubrium percurrit, quam nauigium, quod demonstrandum erat.

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Quod si a puncto b, rectam lineam utriusque ducamus hk, ad remi mensuram, rectos facientem angulos cum bd, rectamque az secantem in i, manifeste intelligemus ipsum rectam az constare ex ai & iz, quorum prior respondet curae ah, quae motu proprio manubrii descripta est: posterior vero æqualis est rectæ bd, quæ motu nauigii decursa est.

Propositio secunda.

Si remi manubrium motu proprio, & nauigium æqualia spatia pertransierint, fieri non poterit, ut palmula moueatur: sed veluti centrum immota manebit.

Esto iterum remus ac, manubrium a, scalmus b: tantum autem spatium conficiat nauigium, quantum motu proprio a. Dico quod c, remi palmula immota manebit. Nam si a loco suo dimota fuerit: spatium igitur permeet cd ad posteriora: quo quidem decurso remus ac, positionem rectitudinis habeat fd. Scalmus itaque b, translatus erit in g.
Excitetur autem a puncto b in utramque partem linea e b r, ad rectos angulos super bg, & a puncto a, recta ah super df: itemque a puncto c, recta ce super er, ipsarum vero rectarum linearum er & ah, sectio sit in k, sed ce & df, sit in z: & quoniam ak, id spatium est quod motu proprio remi manubrium permeaut, curullinoe enim respondet ar, recta autem bg, id spatium est, quod nauigium confection: ipsae igitur rectae lineae ak & bg, æquales erunt. Atqui in duobus æquiangulis triangulis ebc & bak, vel per 26. propositiones primi Euclidis, vel per 4. sexti, æquales esse concludes ak & ec rectas lineas: quapropter æqualis erit ec rectæ bg, per communem sententiam: eodem autem bg, æqualis est ez, in parallelogrammo per 34. propositionem ipsius primi libri: æqualis igitur erit recta ez rectæ ec, pars toti: quod est impossibile, & propterea immota manebit palmula c, quod erat a nobis ostendendum.

Idem alter demonstrabis ostensoria demonstratione. Remus in principio motus positionem habeat abc, ducatur a puncto c, in quo remi palmula, recta linea cg: rectos efficiens angulos in puncto g, cum ea recta linea per quam ad motum nauigii scalmus b mouetur, ipsa deinde recta linea cg, producatur usque ad e, ut sit ge æqualis ab.

Rursus a puncto b, super bg, ad rectos angulos recta linea excitetur kbf, in quam veniant ex a & c, perpendicularares ak & cf. Et quia ipsæ eodem rectæ lineæ ak & cf, æquales sunt per 26. primi Euclidis, ipsi autem fc recta bg, est æqualis in parallelogrammo: igitur æqualis erit bg, per communem sententiam. Atqui tantum spatium conficit b, quantum nauigium, ipsum vero nauigium quantum a, motu proprio per hypothesim: conficit autem spatium ak: conficiet igitur b spatium bg, & quia anguli ad g recti sunt: idcirco cum scalmus peruererit ad g, habebit remus ac, rectitudinis situm ec, in quo loco illius remigrationis finis erit.

Sic igitur palmula c, a loco suo dimota non fuit, quod demonstrandum erat. Caeterum aduertendum est rectam gc, minorem esse bc, remi dimidio: sit autem earum differentia ct: igitur quo tempore Scalum b transfertur in g, excurrat palmula c, in ipsam longitudinem ct, sed neque ad posteriora, neque ad anteriora mouebitur: hoc enim solum demonstrare volumus. Fieri tamen posse non dubitamus, ut aliquando tam dissimili impulsione, tamque inæquali motu feratur nauigium, ut remi palmula aliquidisper in adversum moueatur, sed confestim ad priorem locum remebat. Neque prius, aut posterius, scalmus perueniet ad g, quam ipsa palmula se appellat ad ct, quasi digressa non fuisset a loco.
Propositionis conuersio.

Huius propositionis conversionem demonstrabis, nempe si remi palmula dimota non fuerit a loco suo, ibique tamdu persistat, donec remus situm rectitudinis obtineat, tantum spatiurn confeceret manubrium motu proprio, quantum nauigium. Recta enim cf æqualis est ak per 26. primi: æqualis etiam bg per 34. ipsius primi libri: igitur ak & bg, æquales erunt per communem sententiam.

Proposicio tertia.

Si remi manubrium motu proprio duplum confecerit spatiurn, quanm nauigium tantum prouehetur ea remigatione nauigium, quantum palmula retrocresserit.

Remus enim incipiente motu positionem habeat ac, desinente vero rectitudinis situm fg: Scalms igitur b propter nauigii motum, spatiurn conficiet bd. Excitetur a puncto b, in utramque partem perpendicularis ez, in quam veniant a punctis a & c, ad rectos angulos rectæ lineæ ae & cz: spatium autem ae, a manubrio decursum motu proprio spatii bd, duplum sit: recta vero linea ch, curæ repondeat cg, quæ a remi palmula descripta est. Dico ipsas rectas lineas bd & ch, æquales esse. Nam in duobus triangulis bae & cbz, due rectæ lineæ ae & cz, æquales sunt. In parallelogrammo autem bh, due bd & hz æquales, atqui rectæ ae, dupla est rectæ bd, per hypothesim: dupla est igitur & cz rectæ hz: qua propter ch & hz, æquales erunt. Due igitur ch & bd æquales, per communem sententiam: Et quia nauigium tantum spatiurn decurrit semper, quantum scalms: si igitur remi manubrium motu proprio duplum confecerit spatium quam nauigium, tanto prouehetur nauigium, quantum palmula retrocresserit, quod demonstrandum erat.
Proposito quarta.

Si nauigium minus spatium decurrat, quam remi manubrium, sed supra dimidium, magis prouehetur, quam palmula retrocedat: si vero citra dimidium, minus.


Corollarium.

Ex hac & præcedenti infertur, quod si remi manubrium motu proprio maius spatium decurrat, quam nauigium, siue id sit duplum, siue minus duplo, siue maius duplo, spatium quod nauigium interim decurrat ad anteriora, & quod palmula remi in contrarium simul juncta, ei quod ipsum remi manubrium motu proprio conficit, æqualia erunt. Semper enim bd, æqualis est hz: tota vero cz, quæ æqualis est ae, ex suis constat partibus ch & hz.

Propositionis conversio.

Si nauigium longius progradiatur, quam remi palmula retrocedat, spatium conficiet plusquam dimidium eius quod motu proprio remi manubrium decurrat: si minus, citra dimidium.

Huius demonstratio ex supradictis facile colligi poterit.

Proposito quinta.

Si celerius feratur nauigium, quam remi manubrium, mouebitur palmula in ulteriora nilque unquam retrocedet, idque spatium decurrat, quo nauigii motus motum manubrii superat.

Habeat enim remus incipiente motu propositionem ae: desinente vero situm rectitudinis fg. Scalmus igitur b, propter nauigii motum translatus, erit in d. Sit itaque spatium bd, maius quam ah, a remi manubrio motu proprio decursum: sic enim celerius dicetur ferri nauigium, quam manubrium.

Dico quod palmalam c, in ulteriora mouebitur. Nam cum Scalmus b, proiectus fuerit in d: translatæ erit ipsa palmula c ubi g, in rectitudinis situ, spatiumque conficiet cg curulineum cui respondet ek: mouebitur igitur palmula in ulteriora. Nihil autem unquam retrocedere, ostendetur in hunc modum. Eadem enim celeritate mouentur a, in h & c, versus i, circa scammum. Atqui per hypothesin celerius fertur nauigium, quam a in h: celerius igitur ipsum nauigium fertur, quam c versus i. Sed mouentur idem c, ipsa nauigii celeritate versus k: celerius igitur ferretur c ad k,
quam ad i: quapropter nihil unquam retrocedet ipsum c, imo vero in ulteriora progredietur, spatiumque decurret ck, quod quidem relinquitur detracto ic ex ik. Si enim remi palmula tota ipsa nauigii celeritate moueretur, ultra k progrederetur, cum b perueniret ad d: sed retrahitur interim, propter eum motum qui fit circa b. Sic igitur palmulae celeritate quae a motu nauigii pronenit retardata, decursum spatium erit ck. Videtur autem solo remorum impulsu hoc fieri non posse, sed alia insuper virtute impellente opus esse.

Ex his Theorematis liquet, quam incerta interroget Aristoteles, & quam inscite respondeat. Nam non continuo si nauigium in anterio rora mouetur, remi palmula retrocedet, neque etiam si retrocedat, minus spatium transmittit in contrarium, quam nauigium progrediatur. Demonstrant hoc secunda & tertia propositio. Remi vero manubrium motu proprio qui circa scalnum fit, & una nauigii motu maius spatium conficit quam nauigium: solo autem proprio motu, si contingat tanta spatium conficere, quantum nauigium, fieri non poterit ut palmula moueratur. Frustra igitur conatur in uniuersum demonstrare remi manubrium maius spatium decurrere, quam palmulum in contrarium. Praeterea quando nauigium longius progrediatur, quam remi palmula regrediatur, minus spatium decurrit quam manubrium: igitur non aequale. Et proinde constat neque veritatem in proposito, neque demonstrationem in iis quae congerit reperiri.

FINIS